RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2016

SECOND YEAR [BATCH 2015-18]

Date : 23/12/2016 **MATHEMATICS [General]**

Time : 11 am – 2 pm **Paper : III** Full Marks : 75

[Use a separate Answer Book for each Group]

Group – A

(*Answer any four questions*) [4×5]

1. Find the foot of the perpendicular drawn from the point P(1, 8, 4) on the straight line joining the points A(0, –11, 4) and B(2, –3, 1). [5]

2. Find the equation of the plane bisecting the angle between the planes $3x-6y+2z+5=0$ and $4x-12y+3z-3=0$ which contains the origin. [5]

3. Prove that the straight lines x = nz + a, y = mz + b and x = z + 1, y = z + 2 are coplanar, if $(a-1)(m-1)=(b-2)(n-1)$. [5]

4. Find the shortest distance between the straight lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$. Find also the equation of the line of shortest distance. [3+2]

5. Find the values of 'a' for which the plane $x+y+z=a$ is a tangent plane to the sphere $x^2+y^2+z^2=27$. [5]

6. Find the equation of the right circular cone whose vertex is at the origin, axis is the z-axis and which passes through the point $(3,-4,6)$. [5]

Group – B

(*Answer any four questions*) [4×10]

7. a) Food X contains 5 units of vitamin A and 6 units of vitamin B per gram and costs 20 p/gm. Food Y contains 8 units of vitamin A and 10 units of vitamin B per gram and costs 30p/gm. The daily requirements of A and B are atleast 80 and 100 units respectively. Formulate the above as a linear programming problem to minimize the cost. [4]

   b) Make a graphical representation of the set of constraints in the following L.P.P and solve it.

   Maximize $z = 2x_1 + x_2$

   Subject to $4x_1+3x_2 \le 12$

   $4x_1+x_2 \le 8$

   $4x_1-x_2 \le 8$

   and $x_1, x_2 \ge 0$ [6]

8. a) Define Basic feasible solution of a system of m linear equations with n unknowns. [3]

   b) Find all the basic feasible solutions of the system

   $2x_1+6x_2+2x_3+x_4=3$

   $6x_1+4x_2+4x_3+6x_4=2$. [7]

9. a) Define convex set and extreme point of a convex set. [3]

   b) If x_1, x_2 be real, show that the set $S=\{(x_1,x_2): 2x_1^2+x_2^2 \le 6\}$ is a convex set in E^2. Which are the extreme points? [7]

10. a) Prove that a hyperplane is a convex set. [3]

b) $x_1 = 2,\ x_2 = 3,\ x_3 = 1$ is a feasible solution of the L.P.P.

Maximize $z = x_1 + 2x_2 + 4x_3$

subject to $2x_1 + x_2 + 4x_3 = 11$

$3x_1 + x_2 + 5x_3 = 14$

$x_1, x_2, x_3 \geq 0$

Reduce the solution to a basic feasible solution. [7]

11. a) Use Charnes' Big-M method to solve the following L.P.P.

Maximize $z = 2x_1 - 3x_2$

subject to $-x_1 + x_2 \geq -2$

$5x_1 + 4x_2 \leq 46$

$7x_1 + 2x_2 \geq 32$

$x_1, x_2 \geq 0$ [7]

b) Find the dual of the following L.P.P

Maximize $z = 2x_1 + 3x_2 - x_3$

subject to $x_1 - x_2 + x_3 \geq 5$

$2x_1 - x_2 + 3x_3 \leq 6$

$3x_1 - 2x_2 + x_3 \leq 8$

and $x_1, x_2, x_3 \geq 0$ [3]

12. a) Find an initial basic feasible solution of the transportation problem [3]

	D_1	D_2	D_3	D_4	D_5	a_i
O_1	2	11	10	3	7	4
O_2	1	4	7	2	1	8
O_3	3	9	4	8	12	9
b_j	3	3	6	5	6	

b) A company has four machines on which to do three jobs. Each job can be assigned to one and only machine. The cost of each job on each machine is given in the adjacent tableau. What are the job assignment which will minimize the cost? [7]

		MACHINE			
		W	X	Y	Z
JOBS	A	18	24	28	32
	B	8	13	17	19
	C	10	15	19	22

13. a) Determine the optimal basic solution of the following transportation problem : [7]

	D_1	D_2	D_3	D_4	a_i
O_1	2	3	11	7	6
O_2	1	0	6	1	1
O_3	5	8	15	9	10
b_j	7	5	3	2	

b) Show that the feasible solution (1, 0, 1, 6) of the system

$x_1 + x_2 + x_3 = 2$

$x_1 - x_2 + x_3 = 2$

$2x_1 + 3x_2 + 4x_3 - x_4 = 0$

is not basic. [3]

Group – C

(*Answer <u>any three</u> questions*) [3×5]

14. Establish the following relations between the shift operator 'E' and the difference operator 'Δ' :

i) $E \cdot \Delta = \Delta \cdot E$

ii) $D = \frac{1}{h}\left[\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \ldots\right], D \equiv \frac{d}{dx}$ [2+3]

15. Evaluate f(0·33), using the following table : [5]

x	:	0·3	0·4	0·5	0·6	0·7
f(x)	:	0·6179	0·6554	0·6915	0·7257	1·7580

16. Using Lagrange's interpolation formula, calculate from the following table, the value of y at x = 102

x	:	93·0	96·2	100·0	104·2	108·7
y	:	11·38	12·80	14·70	17·07	19·91

[5]

17. Calculate by Simpson's ⅓rd rule, the value of the integral $\int_0^1 \frac{x\,dx}{1+x}$, correct upto three significant figures by taking six intervals. [5]

18. Calculate the positive root of the equation $x^2 + 2x - 2 = 0$, correct upto two significant figures by the Newton-Raphson method. [5]

———— × ————